Investigation the US Financial Crisis Contagion to Istanbul Stock Exchange: An Application of Bivariate GARCH Models

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Abstract
This paper investigates the contagion effect between stock markets of US and Turkey due to Global Financial Crisis (2007-2009). We apply four Bivariate GARCH models including Scalar-BEKK and Diagonal-BEKK with variance targeting introduced by Engle and Mezrich (1996), Integrated Dynamic Conditional Correlation (DCC-INT) and Mean Reverting DCC (DCC-MR) models introduced by Engle (2002) to estimate dynamic correlation and investigate recent financial crisis from US to Istanbul Stock Exchange (ISE). We use weekly S&P-500 and ISE-XU100 data for the time period of 2000-2010. Our results show that in all of these four models, the time-varying correlation between US and ISE is increasing during financial crisis period (2007-2009). The empirical evidence confirms the existence of contagion effect from the US to the ISE, for the examined financial crisis.

Keywords: Time-varying correlation, contagion effect, financial crisis, multivariate GARCH models, dynamic conditional correlation models.

JEL-Classification: G1, G15

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Introduction

The US financial crisis really started to show its effect in the middle of 2007 into 2008. In this paper we investigate the transmission of recent US financial crisis to Istanbul Stock Exchange. We follow two goals in this paper. First Is there empirical evidence of contagion from the US financial crisis to Istanbul Stock Exchange? Second if so, when did it occur and for how long did it last? One measure to assess the spillover effects and correlation between stock markets is to look at the correlation estimation of stock returns between financial markets time series. In financial econometrics to estimation and analysis the volatility and time-varying correlation between financial time series, multivariate GARCH (MGARCH) models are employed (Brock, 2008). The first MGARCH model for estimate the conditional covariance matrix between asset prices returns is the VECH model that is proposed by Bollerslev, Engle and Wooldridge (1988). A restricted version of VECH model is the Baba-Engle-Kraft-Kroner (BEKK) model defined by Engle and Kroner (1995). This model has a good property that covariance matrix is positive definite by construction. This model can be used in two scalar and diagonal version with variance targeting restriction (Engle and Mezrich, 1998). One of the most simple class of multivariate GARCH models is the constant conditional correlation (CCC-MGARCH) model of Bollerslev (1990). In this model the conditional correlation matrix is time-invariant. This model is very restrictive because in this model is assumed that correlation matrix is constant. Engle(2002) proposed an extension of CCC model that it is called Dynamic Conditional Correlation (DCC) model. This model has
two Mean Reverting (MR-DCC) and Integrated (INT-DCC) version (Engle, 2002). In this paper we use scalar and diagonal BEKK and also DCC-MR and DCC-INT bivariate GARCH model for estimate and analysis the contagion effect of recent US financial crisis to Istanbul stock market. We use S&P-500 and ISE-XU100 indices weekly data. Empirical result of above four models shows contagion effect of recent financial crisis from US to Istanbul Stock Exchange.

This paper is organized as follow. Section 2 presents literature review on market contagion and empirical studies. Section 3 gives the methodology to estimate the time varying correlation. Section 4 presents data analysis and the empirical results. Finally, section 5 provides conclusions.

**Literature review**

The empirical literature on contagion is extensive. Kim and Kim (2011) estimate dynamic conditional correlations of financial asset returns across countries by multivariate GARCH models and analysis spillover effects of the recent US financial crisis on 5 emerging Asian countries. They find a symptom of financial contagion around the collapse of Lehman Brothers in September 2008.

Yin et al (2010) investigates the spillover of financial crisis by studying the dynamics of correlation among eleven Asian and six Latin American stock markets against US stock market. They estimate the time-varying volatility correlation between the regional factor and the US stock market by an asymmetric dynamic conditional correlation model. They find that there is a significant rise in the estimated time-varying correlation in the period from August 2007 to March 2009, suggesting evidence of contagion from the US stock market to these markets.

Dimitris et al (2011) investigates financial contagion in a multivariate time-varying asymmetric framework, focusing on four emerging equity markets, namely Brazil, Russia, India, China (BRIC) and two developed markets (U.S. and U.K.), during five recent financial crises. The empirical evidence confirms a contagion effect from the crisis country to all others, for each of the examined financial crises.

Manolis and Georgios (2011) applies the Dynamic Conditional Correlation (DCC) multivariate GARCH model in order to examine the time-varying conditional correlations to the weekly index returns of seven emerging stock markets of Central and Eastern Europe. The main finding of the present analysis is that there is a statistically significant increase in conditional correlations between the US and the German stock returns and the CEE stock returns, particularly during the 2007-2009 financial crises.

Filis et al (2011) investigated the time-varying correlation between stock market prices and oil prices for oil-importing and oil-exporting countries.

**Correlation estimation**

To estimate the correlation between two variables the covariance matrix is needed. Multivariate GARCH model give the time-varying covariance matrix of financial time series. A most general version of multivariate GARCH models is the VECH-GARCH model introduced by Bollerslev, Engle and Wooldridge (1988). This model specified as:
\( VEC(H_t) = VEC(\Omega) + AVEC(x_{t-1}x_{t-1}') + BVEC(H_{t-1}) \)  

Where VECH denotes the column-stacking operator applied to the upper portion of the symmetric matrix. A, B and \( \Omega \) are matrixes of parameters that must be estimated. The number of parameters that must be estimated in this model are too many when the dimension of model increase. In is necessary to set some restriction on VECH model to decrease the number of parameters.

Engle and Kroner (1995) specified a new class of VECH models that impose some restriction that allows to covariance matrix to be positive definite. This model represented as:

\[ H_t = \Omega + A(x_{t-1}x_{t-1}')A' + BH_{t-1}B' \]

Where \( H_t \) is covariance matrix at t and A, B and \( \Omega = WW' \) are matrixes of parameters that must be estimated. W is a triangular matrix.

One of the important flexible version of BEKK model is the BEKK model with the variance targeting that it is specified in two diagonal and scalar specifications. Engle and Mezrich (1996) proposed BEKK specification with variance targeting restriction that it is expressed as:

\[ VEC(\Omega) = (I - A - B).VEC(S) \]

\[ \Omega = (1 - \alpha - \beta)S \]

In the two above equations S is the unconditional covariance matrix. Where the equation (3) indicated the diagonal BEKK with the variance targeting and the equation(4) presented the scalar BEKK with the variance targeting.
One of the other specifications of multivariate GARCH models that it is proposed by Bollerslev (1990), named constant conditional correlation model. This model specified as:

\[ D_t = \text{Diag}\{\sqrt{h_{i,t}}\} \quad H_t = D_t RD_t \]

Where \( R \) is a correlation matrix that it is constant over time. This assumption cannot be correct. The information is updated over time and the relationship between variables is changed. Then the constant correlation over time cannot to be true.

Engle and Sheppard (2001) introduced a new multivariate GARCH model that it allows \( R \) to be time-varying. The fundamental equations of DCC model represented as:

\[ \begin{align*}
  x_t | \Phi_{t-1} & \approx N(0, D_t R_t D_t) \\
  D_t^2 & = \text{diag}(\omega_i) + \text{diag}(\kappa_i) \circ x_{t-1} x_{t-1}' + \text{diag}(\lambda_i) \circ D_{t-1}^2 \\
  \varepsilon_t & = D_t^{-1} x_t \\
  Q_t & = S \circ (1' - A - B) + A \circ \varepsilon_{t-1} \varepsilon_{t-1}' + B \circ Q_{t-1} \\
  R_t & = \text{diag}(Q_t)^{-1} Q_t \text{diag}(Q_t)^{-1}
\end{align*} \]

Where \( D_t \) is the diagonal variance matrix in time \( t \), also \( S \) is the unconditional covariance matrix. also \( Q_t \), \( R_t \) are the conditional covariance and correlation matrix in time \( t \). Engle (2002) specified two classes of DCC models that they named DCC Mean-Reverting (DCC-MR) and DCC-Integrated (DCC-INT) models. This models are shown in following equations. In equation (7) the matrix version of DCC-MR model is expressed. Where \( \alpha \) and \( \beta \) are the parameters of DCC-MR model that can be estimated by maximization of log-likelihood function (Engle, 2002).

\[ Q_t = S(1 - \alpha - \beta) + \alpha(\varepsilon_{t-1} \varepsilon_{t-1}') + \beta(Q_{t-1}) \]
The scalar and matrix version of DCC-INT models are exhibited in equations (8) and (9) respectively.

\[ \rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}} \quad q_{ij,t} = (1-\lambda)(\epsilon_{i,t-1} - \epsilon_{j,t-1}) + \lambda(q_{ij,t-1}) \] (8)

\[ Q_t = (1-\lambda)(\epsilon_{t-1}'\epsilon_{t-1}) + \lambda Q_{t-1} \] (9)

As proposed by Engle (2002), the DCC model can be estimated by using a two-stage approach to maximizing the log-likelihood function. The DCC model has advantage over other estimation models such as VECH and BEKK in the number of estimating parameters. Thus in this paper we use this model to investigating the correlation between variables.

**Data description**

The data consists of weekly stock indices from 2000.1.7 to 2010.9.10 for a total of 558 observations. The sample consists of two countries USA (S&P-500 index) and Turkey (ISE-XU100 index). Figures 1 and 2 plot S&P-500 and ISE-XU100 indices over time respectively. Weekly stock return for both indices are calculated as

\[ R_t = 100 \log(P_t / P_{t-1}) \]

where \( P_t \) is the price level of each stock index at time \( t \). Table 1 contains some descriptive statistics of stock returns time series of data. This result suggests that the stock return series may not be normally distributed.
Table 1. Descriptive statistics of stock returns

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P</th>
<th>ISE-XU100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.046985</td>
<td>0.334443</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2.780285</td>
<td>5.571259</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.841346</td>
<td>-0.144004</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>9.654660</td>
<td>6.85852</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>1093.483</td>
<td>352.2496</td>
</tr>
</tbody>
</table>

Empirical results

We employed four different models to estimate the correlation. The models and their specification are:
1. Scalar-BEKK with variance targeting that specified in equation (4).
2. Diagonal-BEKK with variance targeting in equation (3).
3. DCC-INT model that it is presented in equation (8) and (9).
4. DCC-MR model that it is specified in equations (7).
These four specification of multivariate GARCH models have more flexibility than the other models such as constant conditional correlation and VECH models. In this section, we report the results that obtained from
the application of the multivariate GARCH models to estimation time-varying correlation between US and Turkey stock markets.

A plot of GARCH volatilities of these two series resulted from estimation the univariate GARCH model is apparent in figure 2, 3.

![Figure 3](image1.png) ![Figure 4](image2.png)

**Figure 3.** S&P stock return volatility

**Figure 4.** ISE-XU100 stock return volatility

In this figures it is clear that volatility of both S&P and ISE-XU100 stock returns is increasing in 2007-2009 period. This is due to financial crisis of this period.

The DCC parameter estimation for Integrated and Mean reverting model are exhibited in table 2, 3. The obtained coefficients were statistically significant at 5%.

<table>
<thead>
<tr>
<th>variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lambda-DCC</td>
<td>0.032642</td>
<td>0.011526</td>
<td>2.832049</td>
<td>0.0046</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>AIC</td>
<td>3.670011</td>
<td>3.673042</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SBC</td>
<td>3.677771</td>
<td>3.677771</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. DCC-INT model parameter estimation
Table 3. DCC-MR model parameter estimation

<table>
<thead>
<tr>
<th>variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha-DCC</td>
<td>0.033874</td>
<td>0.015025</td>
<td>2.254426</td>
<td>0.0242</td>
</tr>
<tr>
<td>Beta-DCC</td>
<td>0.962660</td>
<td>0.018352</td>
<td>52.45393</td>
<td>0.0000</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-1020.802</td>
<td>3.672537</td>
<td>3.688058</td>
<td>3.678599</td>
</tr>
</tbody>
</table>

In table 4 and 5 the parameter estimation of Diagonal-BEKK and Scalar-BEKK models are expressed. All obtained coefficients were statistically significant at 5%.

Table 4. Diagonal-BEKK model parameter estimation

<table>
<thead>
<tr>
<th>variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>0.310396</td>
<td>0.017875</td>
<td>17.36464</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(2)</td>
<td>0.943475</td>
<td>0.007377</td>
<td>127.8962</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(3)</td>
<td>0.220986</td>
<td>0.013848</td>
<td>15.95760</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(4)</td>
<td>0.973222</td>
<td>0.003669</td>
<td>265.2558</td>
<td>0.0000</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-7558.521</td>
<td>27.15448</td>
<td>27.18552</td>
<td>27.16660</td>
</tr>
</tbody>
</table>

Table 5. Scalar-BEKK model parameter estimation

<table>
<thead>
<tr>
<th>variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>0.071882</td>
<td>0.005906</td>
<td>12.17108</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(2)</td>
<td>0.919182</td>
<td>0.007246</td>
<td>126.8567</td>
<td>0.0000</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-7560.468</td>
<td>27.15428</td>
<td>27.16980</td>
<td>27.16035</td>
</tr>
</tbody>
</table>

In figures 5, 6 the time-varying correlation between S&P and ISE-XU100 stock returns is exhibited based on DCC-INT and DCC-MR estimated models. Also figure 7,8 shows dynamic correlation of these time series based on computed Diagonal-BEKK and Scalar-BEKK models.
Based on all four models that have been estimated, the mean of dynamic correlation of stock price returns of US and Turkey stock markets had been increasing in 2007-2009 period. Indeed in financial crisis period the mean of correlation is higher than pre-crisis period during the 2007-2009 periods. We observe that correlation among US and Turkey stock market is increased. In the pre-crisis period the mean of correlation is 0.1 but in the 2007-2009 periods the mean of correlation is 0.5 based on the time-
varying correlation that it is estimated by every four models that has been estimated. This shows evidences of financial crisis contagion from US to Istanbul stock market. The results also show that the financial crisis is quickly transferred from US to Turkey stock market, as contagion effect can be seen from start of 2008.

Conclusion
This paper investigation time-varying correlation between USA and Turkey stock markets. The conditional correlation is estimated following DCC-MR and DCC-INT (Engle, 2002) and also Scalar-BEKK and Diagonal-BEKK with variance targeting (Engle and Kroner, 1995, Engle and Mezrich, 1996) models. We use maximum-likelihood estimation method to estimate the parameters of these four models. From obtained empirical results it is clear that the dynamic correlation between stock prices returns is increasing during financial crisis period. This result show evidences of financial spillover effect from US to Istanbul stock market. These results contain very important information for investors that want to invest in these stock markets such as portfolio diversification policies.

References
